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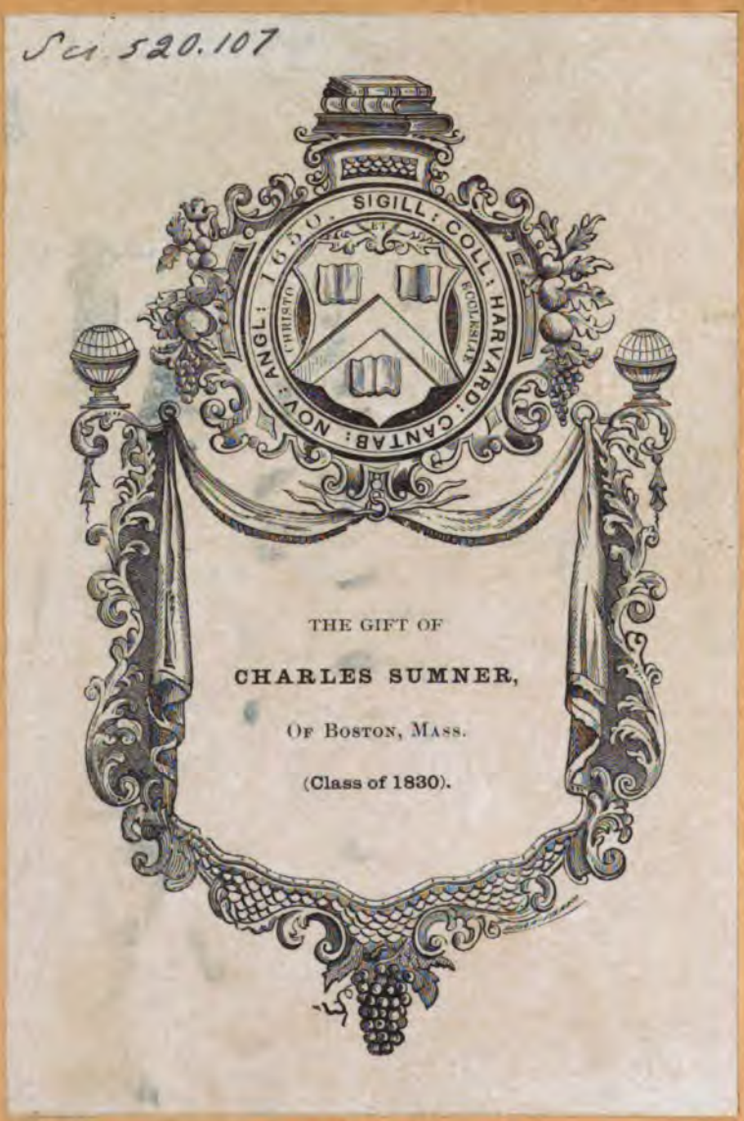

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WASHINGTON OBSERVATIONS FOR 1870.—APPENDIX II.

REPORTS

ON

OBSERVATIONS OF ENCKE'S COMET

DURING ITS RETURN IN 1871.

BY

ASAPH HALL AND WM. HARKNESS,

PROFESSORS OF MATHEMATICS, U. S. NAVY.

PREPARED AT THE U. S. NAVAL OBSERVATORY

BY ORDER OF

REAR-ADMIRAL B. F. SANDS, U. S. N.,

SUPERINTENDENT.

WASHINGTON:

GOVERNMENT PRINTING OFFICE.

1872.

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Encke's Comet as seen in the 9½-inch Equatorial on the nights of October 17, November 17, December 1, and December 2.

REPORT

OF

PROFESSOR ASAPH HALL, U. S. N.

REPORT OF PROFESSOR HALL, U. S. N.

UNITED STATES NAVAL OBSERVATORY,
Washington, January 15, 1872.

SIR: The motion of Encke's comet around the sun has given rise to many interesting investigations, and the results obtained are such as will doubtless lead, sooner or later, to a new and complete discussion of the observations, together with a new investigation of the theory of the comet. The following observations were made during the favorable return of the comet in 1871. They are divided into two parts. The first part consists of determinations of position, and in order to give a clear idea of the degree of accuracy attained, these observations are given in detail. The second part comprises the notes on the appearance of the comet in the telescope, and to these are added four drawings, made from sketches of the comet on October 17, November 17, and December 1 and 2. These drawings are intended to illustrate the changes of form that the comet underwent as it approached the sun.

§ 1.

The following observations of the position of the comet were made with the filar micrometer of the Equatorial of the Naval Observatory. The reticule of this micrometer consists of three parallel wires, with equatorial intervals of $12^s.4$, for observations of the difference of right ascension; and perpendicular to these are five parallel wires used for measuring differences of declination. The differences of right ascension were observed by means of a chronograph, the pen of which is worked by a sidereal clock. This clock is situated on the first floor of the Observatory, and for the sake of convenience, the time of the observation was taken from a sidereal chronometer, the second of which was compared with the clock on the chronograph-sheet, at the beginning and end of each observation. No attention is paid to the clock further than to keep its rate so small that the effect of the rate on the observed difference of right ascension is always insensible. In reading off the chronograph-sheet, the zero is assumed to be at the second of the clock nearest the zero second of the chronometer. This assumption introduces an error into the determination of the time of the observation, which will vary from $0^s.0$ to $0^s.5$; and, in the case of a comet or swiftly-moving body, a correction for the error of the zero is combined with the correction of the chronometer. The chronometer was compared on each night with the Kessels clock, the standard clock of the Observatory.

In what follows $\Delta\alpha$ and $\Delta\delta$ are the observed differences of right ascension and declination at the corresponding chronometer-time. The value of $\Delta\alpha$ is generally the

mean of three wires, except in cases when the star and comet were so situated that it was inconvenient to observe all the wires, when only two wires were used. The value of $\Delta\delta$ is the result of a single bisection with the micrometer wire. The quantities under the head *Red.* are the reductions of the observed $\Delta\alpha$ and $\Delta\delta$ to the assumed chronometer-time of the observation, which is designated by c. These reductions have been computed from the ephemeris of the comet given by S. von Glasenapp, Astronomische Nachrichten, No. 1854, after the ephemeris had been corrected so as to represent very nearly the apparent motion of the comet. $\Delta_o\alpha$ and $\Delta_o\delta$ are the values of $\Delta\alpha$ and $\Delta\delta$ reduced to the chronometer time c; Δc is the correction of the chronometer on sidereal time; and $\Delta\rho$ is the correction for differential refraction. $r(\alpha)$ denotes the probable error of a difference of right ascension depending on an observation over a single wire, and $r(\delta)$ the probable error of a single bisection in declination. In the statement of the number of comparisons, the first number indicates the number of wires observed in right ascension, and the last the number of bisections in declination.

The positions of the stars of comparison are those which have been computed from the catalogues, no "systematic corrections" having been applied. The positions have been brought forward to 1871.0 by means of Peter's value of the constant of precession, and the reductions of the stars to apparent place have been computed from the quantities given in the American Ephemeris. These reductions are given for each date below the adopted mean position of the star.

It should be stated that in a few of the earlier observations, or until November 17, the point observed was the center of the figure of the comet, while in the later observations the nucleus of the comet was observed.

The objective of the Equatorial has an aperture of 9.6 inches, and a magnifying power of 132 was used in all the observations.

November 2.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_o\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_o\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
21 53 40.8	− 0 48.75	− 1.30	50.05	22 2 48.0	− 167.2	+ 7.2	− 160.0
55 18.7	− 0 48.75	− 0.63	49.38	4 50.0	− 181.3	+ 9.7	− 171.6
56 55.6	− 0 51.10	+ 0.03	51.07	6 27.0	− 189.4	+ 11.6	− 177.8
58 25.0	− 0 51.20	+ 0.64	50.56	8 9.0	− 188.2	+ 13.7	− 174.5
59 57.2	− 0 51.45	+ 1.27	50.18	9 55.0	− 189.4	+ 15.8	− 173.6

No. of Comparisons, (10.5.)

h. m. s.	m. s. s.	′ " "
c = 21 56 51.5	$\Delta_o\alpha$ = − 0 50.25 ± 0.19	$\Delta_o\delta$ = − 2 51.5 ± 2.1
Δc = − 4 28.3	$\Delta\rho$ = 0.00	$\Delta\rho$ = − 0.0
	$r(\alpha)$ = ± 0ˢ.61	$r(\delta)$ = ± 4″.6

Star of Comparison, a.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	22 37 4.28	+ 37 7 41.1
Washington observations, (2) .	4.06	41.3
Adopted	22 37 4.13	+ 37 7 41.2
Reduction	+ 1.95	+ 23.9

NOVEMBER 6.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 34 8.4	+ 0 49.73	− 6.56	43.17	23 43 46.0	− 442.4	− 11.7	− 454.1
36 10.6	+ 0 50.67	− 5.71	44.96	46 0.0	− 443.1	− 7.4	− 450.5
37 59.4	+ 0 50.23	− 4.96	45.27	48 15.0	− 459.6	− 3.1	− 462.7
39 41.0	+ 0 49.33	− 4.25	45.08	50 27.0	− 454.6	+ 1.2	− 453.4
41 28.7	+ 0 48.47	− 3.50	44.97	53 36.0	− 455.2	+ 5.3	− 449.9
58 49.3	+ 0 40.76	+ 3.75	44.51	56 10.0	− 468.2	+ 12.1	− 456.1
23 0 21.0	+ 0 40.00	+ 4.39	44.39				
1 45.3	+ 0 39.40	+ 4.98	44.38				
3 18.3	+ 0 38.30	+ 5.63	43.93				
4 45.3	+ 0 38.63	+ 6.23	44.86				

No. of Comparisons, (30.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 49\ 50.7$	$\Delta_0\alpha = +\ 0\ 44.55 \pm 0.15$	$\Delta_0\delta = -\ 7\ 34.4 \pm 1.3$
$\Delta c = -\ 4\ 34.3$	$\Delta\rho =\ 0.00$	$\Delta\rho = -\ 0.0$
	$r(\alpha) = \pm\ 0^s.79$	$r(\delta) = \pm\ 3''.1$

Star of Comparison, b.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	21 55 23.61	+ 34 40 54.7
Reduction	+ 1.54	+ 23.4

NOVEMBER 7.

Chron. Time.			Δa	Red.	$\Delta_0 a$	Chron. Time.			$\Delta \delta$	Red.	$\Delta_0 \delta$
h.	m.	s.	m. s.	s.	s.	h.	m.	s.	"	"	"
22	23	23.2	− 0 30.00	− 4.79	34.79	22	30	28.0	− 60.9	− 9.2	− 70.1
	24	55.4	− 0 28.90	− 4.15	33.05		34	11.0	− 75.7	− 1.4	− 77.1
	26	13.0	− 0 29,85	− 3.61	33.46		35	40.0	− 73.9	+ 1.7	− 72.2
	27	28.3	− 0 30.10	− 3.08	33.18		37	18.0	− 68.2	+ 5.1	− 63.1
	28	41.3	− 0 29.85	− 2.58	32.43		38	49.0	− 69.0	+ 8.3	− 60.7
	40	54.7	− 0 35.95	+ 2.52	33.43						
	42	14.7	− 0 37.70	+ 3.07	34.63						
	43	36.1	− 0 36.20	+ 3.64	32.56						
	44	57.1	− 0 37.55	+ 4.20	33.35						
	46	19.2	− 0 39.45	+ 4.77	34.68						

No. of Comparisons, (20.5.)

h. m. s.	m. s. s.	′ ″ ″
c = 22 34 52.3	$\Delta_0 a$ = − 0 33.56 ± 0.19	$\Delta_0 \delta$ = − 1 8.6 ± 2.0
Δc = − 4 35.4	$\Delta \rho$ = 0.00	$\Delta \rho$ = − 0.0
	$r(a) = \pm 0^s.83$	$r(\delta) = \pm 4''.5$

Star of Comparison, c.

Authority.	a	δ
	h. m. s.	° ′ ″
Bessel	21 46 47.91	+ 33 46 30.2
Reduction	+ 1.46	+ 23.1

NOVEMBER 8.

Chron. Time.			Δa	Red.	$_0 a$	Chron. Time.			$\Delta \delta$	Red.	$\Delta_0 \delta$
h.	m.	s.	m. s.	s.	s.	h.	m.	s.	"	"	"
22	56	54.0	+ 0 21.45	− 4.31	17.14	23	4	19.5	+ 275.4	− 6.7	+ 268.7
	58	5.4	+ 0 22.40	− 3.82	18.58		6	7.1	+ 273.4	− 2.7	+ 270.7
	59	38.2	+ 0 21.65	− 3.18	18.47		9	26.3	+ 254.3	+ 4.8	+ 259.1
23	0	55.1	+ 0 21.20	− 2.65	18.55		11	15.1	+ 257.5	+ 8.9	+ 266.4
	1	48.8	+ 0 19.50	− 2.28	17.24		12	50.3	+ 244.7	+ 12.5	+ 257.2
	15	18.5	+ 0 14.50	+ 3.33	17.83						
	16	33.8	+ 0 14.45	+ 3.84	18.29						
	17	38.2	+ 0 13.30	+ 4.28	17.58						
	18	52.6	+ 0 13.15	+ 4.79	17.94						

No. of Comparisons, (18.5.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 7\ 18.3$	$\Delta_0 a = +\ 0\ 17.96 \pm 0.12$	$\Delta_0 \delta = +\ 4\ 24.4 \pm 1.8$
$\Delta c = -\ 4\ 36.2$	$\Delta \rho =\ 0.00$	$\Delta \rho = +\ 0.1$
	$r(a) = \pm\ 0^s.52$	$r(\delta) = \pm\ 4''.0$

Star of Comparison, d.

Authority.	a	δ
	h. m. s.	° ′ ″
Washington observations, (3) .	21 35 47.62	+ 32 47 16.3
Reduction	+ 1.36	+ 22.7

NOVEMBER 10.

Chron. Time.	Δa	Red.	$\Delta_0 a$	Chron. Time.	$\Delta \delta$	Red.	$\Delta_0 \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 13 10.3	− 2 50.00	− 5.33	55.33	22 13 10.3	+ 146.9	− 33.8	+ 113.1
17 48.4	− 2 49.05	− 3.45	52.50	17 48.4	+ 124.6	− 21.9	+ 102.7
21 54.9	− 2 49.95	− 1.79	51.74	21 54.9	+ 132.8	− 11.3	+ 121.5
26 7.3	− 2 52.75	− 0.08	52.83	26 7.3	+ 129.6	− 0.5	+ 129.1
30 45.8	− 2 50.75	+ 1.80	48.95	30 45.8	+ 102.8	+ 11.4	+ 114.2
35 5.2	− 2 55.10	+ 3.55	51.55	35 5.2	+ 92.6	+ 22.5	+ 115.1
39 23.0	− 2 56.55	+ 5.30	51.25	39 23.0	+ 82.0	+ 33.6	+ 115.6

No. of Comparisons, (14.7.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 26\ 19.3$	$\Delta_0 a = -\ 2\ 52.02 \pm 0.49$	$\Delta_0 \delta = +\ 1\ 55.9 \pm 2.1$
$\Delta c = -\ 4\ 38.7$	$\Delta \rho =\ 0.00$	$\Delta \rho = +\ 0.0$
	$r(a) = \pm\ 1^s.46$	$r(\delta) = \pm\ 5''.4$

Star of Comparison, e.

Authority.	a	δ
	h. m. s.	° ′ ″
Bessel	21 19 38.87	+ 30 55 13.2
Reduction	+ 1.21	+21.7

NOVEMBER 11.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_o\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_o\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
22 11 57.0	+ 3 32.03	− 7.41	24.62	22 11 57.0	+ 196.5	− 50.2	+ 146.3
18 43.7	+ 3 29.45	− 4.70	24.75	18 43.7	+ 192.9	− 31.9	+ 161.0
24 34.3	+ 3 26.13	− 2.36	23.77	24 34.3	+ 176.5	− 16.0	+ 160.5
32 1.8	+ 3 22.20	+ 0.62	22.82	32 1.8	+ 151.5	+ 4.2	+ 155.7
36 56.6	+ 3 21.23	+ 2.58	23.81	36 56.6	+ 140.8	+ 17.5	+ 158.3
41 38.2	+ 3 19.03	+ 4.46	23.49	41 38.2	+ 130.4	+ 30.2	+ 160.6
46 17.2	+ 3 17.60	+ 6.32	23.92	46 17.2	+ 114.3	+ 42.8	+ 157.1

No. of Comparisons, (20.7.)

h. m. s.	m. s. s.	′ ″ ″
c = 22 30 29.0	$\Delta_o\alpha$ = + 3 23.88 ± 0.17	$\Delta_o\delta$ = + 2 37.1 ± 1.3
Δc = − 4 40.2	$\Delta\rho$ = 0.00	$\Delta\rho$ = + 0.0
	$r(\alpha) = \pm 0^s.76$	$r(\delta) = \pm 3''.5$

Star of Comparison, f.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	21 3 41.58	+ 29 51 10.9
Reduction	+ 1.08	+ 20.8

NOVEMBER 17.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_o\alpha$	Chron. Time.	$\Delta\delta$	Red.	$_o\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
23 1 57.4	− 1 6.70	− 2.53	9.23	23 1 57.4	+ 65.6	− 23.5	+ 42.1
5 10.5	− 1 7.50	− 1.40	8.90	5 10.5	+ 60.4	− 12.9	+ 47.5
7 52.6	− 1 8.53	− 0.44	8.93	7 52.6	+ 54.9	− 4.1	+ 50.8
10 34.9	− 1 10.40	+ 0.52	9.88	10 34.9	+ 34.0	+ 4.8	+ 38.8
13 15.3	− 1 10.43	+ 1.46	8.97	13 15.3	+ 32.4	+ 13.5	+ 45.9
15 52.3	− 1 11.00	+ 2.39	8.61	15 52.3	+ 29.7	+ 22.1	+ 51.8

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
c = 23 9 7.2	$\Delta_o\alpha$ = − 1 9.09 ± 0.12	$\Delta_o\delta$ = + 0 46.2 ± 1.4
Δc = − 4 49.7	$\Delta\rho$ = 0.00	$\Delta\rho$ = + 0.0
	$r(\alpha) = \pm 0^s.50$	$r(\delta) = \pm 3''.4$

Star of Comparison, g.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	20 13 51.38	+ 22 35 40.7
Reduction	+ 0.74	+16.1

NOVEMBER 25.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 38 57.6	− 2 29.30	− 3.19	32.49	22 38 57.6	+ 291.5	− 36.8	+ 254.7
43 35.0	− 2 30.73	− 1.83	32.56	43 35.0	+ 280.3	− 21.2	+ 259.1
48 6.9	− 2 32.27	− 0.51	32.78	48 6.9	+ 264.4	− 5.9	+ 258.5
52 3.3	− 2 33.70	+ 0.64	33.06	52 3.3	+ 242.2	+ 7.4	+ 249.6
56 1.8	− 2 34.93	+ 1.80	33.13	56 1.8	+ 235.5	+ 20.8	+ 256.3
60 24.0	− 2 36.23	+ 3.08	33.15	60 24.0	+ 220.2	+ 35.6	+ 255.8

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 49\ 51.4$	$\Delta_0\alpha = -2\ 32.86 \pm 0.08$	$\Delta_0\delta = +4\ 15.7 \pm 0.9$
$\Delta c = -\ 5\ 0.6$	$\Delta\rho = 0.00$	$\Delta\rho = +\ 0.1$
	$r(\alpha) = \pm 0^s.35$	$r(\delta) = \pm 2''.3$

Star of Comparison, h.

Anthority.	α	δ
	h. m. s.	° ′ ″
Bessel	19 13 49.17	+ 11 47 51.0
Reduction	+ 0.54	+ 8.9

NOVEMBER 25.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 48 6.9	− 1 3.67	− 1.76	5.43	22 48 6.9	− 211.4	− 20.4	− 231.8
52 3.3	− 1 5.67	− 0.61	6.28	52 3.3	− 231.9	− 7.1	− 239.0
56 1.8	− 1 6.67	+ 0.55	6.12	56 1.8	− 243.7	+ 6.3	− 237.4
60 24.0	− 1 8.03	+ 1.83	6.20	60 24.0	− 254.4	+ 21.1	− 233.3

No. of Comparisons, (12.4.)

	h.	m.	s.		m.	s.	s.	′ ″ ″
c =	22	54	9.0	$\Delta_o\alpha$ = −	1	6.01	± 0.13	$\Delta_o\delta$ = − 3 55.4 ± 1.1
Δc = −		5	0.6	$\Delta\rho$		0.00		$\Delta\rho$ = − 0.1
						$r(\alpha) = \pm 0^s.45$		$r(\delta) = \pm 2''.3$

Star of Comparison, i.

Authority,	α	δ
	h. m. s.	° ′ ″
Bessel	19 12 21.01	+ 11 55 47.8
Reduction	+ 0.50	+ 8.9

NOVEMBER 26.

Chron. Time.			$\Delta\alpha$	Red.	$\Delta_o\delta$	Chron. Time.			$\Delta\delta$	Red.	$\Delta_o\delta$
h.	m.	s.	m. s.	s.	s.	h.	m.	s.	″	″	″
22	21	51.3	− 0 54.77	− 1.81	56.58	22	21	51.3	− 475.7	− 21.3	− 497.0
	24	14.0	− 0 55.80	− 1.13	56.93		24	14.0	− 477.6	− 13.3	− 490.9
	26	49.4	− 0 56.97	− 0.39	57.36		26	49.4	− 497.1	− 4.6	− 501.7
	29	14.2	− 0 57.70	+ 0.30	57.40		29	14.2	− 504.5	+ 4.4	− 500.1
	32	3.5	− 0 58.07	+ 1.10	56.97		32	3.5	− 517.9	+ 12.9	− 505.0
	34	59.7	− 0 58.97	+ 1.94	57.03		34	59.7	− 527.7	+ 22.8	− 504.9

No. of Comparisons, (17.6.)

	h.	m.	s.		m.	s.	s.	′ ″ ″
c =	22	28	12.0	$\Delta_o\alpha$ = −	0	57.05	± 0.08	$\Delta_o\delta$ = − 8 19.9 ± 1.5
Δc = −		5	1.8	$\Delta\rho$ =		0.00		$\Delta\rho$ = − 0.2
						$r(\alpha) = \pm 0^s.35$		$r(\delta) = \pm 3''.6$

Star of Comparison, k.

Authority.	α	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	19 5 24.56	+ 10 41 11.8	1
Lamont	24.84	7.0	1
Adopted	19 5 24.70	+ 10 41 9.4	
Reduction	+ 0.49	+ 8.1	

NOVEMBER 27.

Chron. Time.			Δa	Red.	$\Delta_0 a$	Chron. Time.			$\Delta \delta$	Red.	$\Delta_0 \delta$
h.	m.	s.	m. s.	s.	s.	h.	m.	s.	"	"	"
22	28	27.8	+ 0 4.67	− 2.40	2.27	22	33	19.0	+ 113.1	− 12.3	+ 100.8
	29	9.6	+ 0 4.30	− 2.20	2.10		34	41.9	+ 106.0	− 7.8	+ 98.2
	29	55.0	+ 0 4.30	− 1.99	2.31		36	2.7	+ 105.7	− 3.3	+ 102.4
	30	55.3	+ 0 3.80	− 1.71	2.09		37	37.3	+ 92.9	+ 2.0	+ 94.9
	31	33.2	+ 0 3.90	− 1.53	2.37		39	26.0	+ 89.4	+ 8.0	+ 97.4
	42	25.7	+ 0 0.77	+ 1.51	2.28		41	1.9	+ 86.9	+ 13.3	+ 100.2
	43	13.0	+ 0 0.53	+ 1.73	2.26						
	44	1.9	+ 0 0.60	+ 1.96	2.56						
	44	57.6	+ 0 0.30	+ 2.22	2.52						
	45	38.8	+ 0 0.00	+ 2.41	2.41						

No. of Comparisons, (30.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 37\ 1.8$	$\Delta_0 a = + 0\ 2.32 \pm 0.03$	$\Delta_0 \delta = + 1\ 39.0 \pm 0.7$
$\Delta c = - 5\ 3.9$	$\Delta \rho = 0.00$	$\Delta \rho = + 0.0$
	$r(a) = \pm 0^s.17$	$r(\delta) = \pm 1''.8$

Star of Comparison, l.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 57 36.64	+ 9 10 52.7	1
Lamont	36.44	56.5	1
Lalande	36.64	59.9	½
Adopted.	18 57 36.56	+ 9 10 55.7	
Reduction	+ 0.48	+ 7.2	

NOVEMBER 29.

Chron. Time.			Δa	Red.	$\Delta_0 a$	Chron. Time.			$\Delta \delta$	Red.	$\Delta_0 \delta$
h.	m.	s.	m. s.	s.	s.	h.	m.	s.	"	"	"
22	49	47.8	+ 2 42.13	− 3.08	39.05	22	49	47.8	+ 189.3	− 37.5	+ 151.8
	53	51.3	+ 2 40.70	− 1.99	38.71		53	51.3	+ 174.6	− 24.2	+ 150.4
	59	7.2	+ 2 39.40	− 0.58	38.82		59	7.2	+ 157.9	− 7.1	+ 150.8
23	4	22.2	+ 2 38.33	+ 0.83	39.16	23	4	22.2	+ 135.6	+ 10.0	+ 145.6
	8	20.9	+ 2 37.43	+ 1.89	39.32		8	20.9	+ 122.3	+ 23.0	+ 145.3
	12	14.6	+ 2 36.37	+ 2.94	39.31		12	14.6	+ 112.8	+ 35.7	+ 148.5

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 1\ 17.3$	$\Delta_o a = +2\ 39.06 \pm 0.07$	$\Delta\delta = +2\ 28.7 \pm 0.8$
$\Delta c = -\ 5\ 7.4$	$\Delta\rho = 0.00$	$\Delta\rho = +\ 0.1$
	$r(a) = \pm 0^s.29$	$r(\delta) = \pm 1''.9$

Star of Comparison, m.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 41 46.34	+ 6 31 8.8	1
Lamont	45.84	10.8	1
Schjellerup	46.10	9.4	2
Adopted	18 41 46.10	+ 6 31 9.6	
Reduction	+ 0.46	+ 5.4	

DECEMBER 1.

Chron. Time.	Δa	Red.	$\Delta_o a$	Chron. Time.	δ	Red.	$\Delta_o \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 36 10.3	+ 2 39.25	− 2.63	36.62	22 36 10.3	+ 169.9	− 32.4	+ 137.5
40 6.3	+ 2 37.67	− 1.61	36.06	40 6.3	+ 158.8	− 19.9	+ 138.9
44 16.5	+ 2 37.03	− 0.54	36.49	44 16.5	+ 144.2	− 6.7	+ 137.5
48 13.3	+ 2 35.97	+ 0.48	36.45	48 13.3	+ 134.6	+ 5.9	+ 140.5
52 55.8	+ 2 34.40	+ 1.69	36.09	52 55.8	+ 124.5	+ 20.9	+ 145.4
56 51.0	+ 2 33.83	+ 2.70	36.53	56 51.0	+ 112.2	+ 33.3	+ 145.5

No. of Comparisons, (17.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 46\ 22.2$	$\Delta_o a = +2\ 36.37 \pm 0.07$	$\Delta_o \delta = +2\ 20.9 \pm 1.0$
$\Delta c = -\ 5\ 8.5$	$\Delta\rho = -\ 0.01$	$\Delta\rho = +\ 0.1$
	$r(a) = \pm 0^s.28$	$r(\delta) = \pm 2''.5$

Star of Comparison, n.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Lamont	18 29 18.07	+ 3 57 56.2	1
Reduction	+ 0.46	+ 4.0	

December 2.

Chron. Time.			Δ α		Red.	Δ₀ α	Chron. Time.			Δ δ	Red.	Δ₀ δ
h.	m.	s.	m.	s.	s.	s.	h.	m.	s.	″	″	″
22	39	45.8	— 0	8.63	— 2.11	10.74	22	43	29.0	— 300.2	— 14.6	— 314.8
	40	43.0	— 0	8.73	— 1.87	10.60		44	48.4	— 307.3	— 10.5	— 317.8
	41	40.0	— 0	9.13	— 1.63	10.76		46	1.8	— 313.3	— 6.6	— 319.9
	42	42.0	— 0	9.13	— 1.37	10.50		47	23.6	— 316.6	— 2.3	— 318.9
	51	39.5	— 0	11.35	+ 0.89	10.46		48	48.3	— 318.9	+ 2.1	— 316.8
	52	44.6	— 0	11.80	+ 1.16	10.64		50	7.4	— 325.2	+ 6.2	— 319.0
	53	40.4	— 0	11.95	+ 1.40	10.55						
	54	40.6	— 0	12.05	+ 1.64	10.41						
	55	38.2	— 0	12.60	+ 1.89	10.71						

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 48\ 8.2$	$\Delta_0 \alpha = -0\ 10.60 \pm 0.03$	$\Delta_0 \delta = -5\ 17.9 \pm 0.5$
$\Delta c = -\ 5\ 9.0$	$\Delta p = +\ 0.02$	$\Delta p = -\ 0.3$
	$r(\alpha) = \pm 0^s.13$	$r(\delta) = \pm 1''.2$

Star of Comparison, o.

Authority.	α			δ			Wt.
	h.	m.	s.	°	′	″	
Bessel	18	25	58.43	+ 2	49	59.1	1
Schjellerup			58.50			53.7	2
Lamont			58.55			56.4	1
Adopted	18	25	58.50	+ 2	49	55.7	
Reduction			+ 0.47			+ 3.5	

December 5.

Chron. Time.			Δ α		Red.	Δ₀ α	Chron. Time.			Δ δ	Red.	Δ₀ δ
h.	m.	s.	m.	s.	s.	s.	h.	m.	s.	″	″	″
23	6	8.6	— 1	43.27	— 1.51	44.78	23	6	8.6	+ 207.0	— 19.2	+ 187.8
	9	31.6	— 1	44.10	— 0.71	44.81		9	31.6	+ 184.9	— 9.1	+ 175.8
	12	37.3	— 1	44.67	+ 0.02	44.65		12	37.3	+ 183.5	+ 0.2	+ 183.7
	15	44.2	— 1	45.20	+ 0.75	44.45		15	44.2	+ 172.0	+ 9.5	+ 181.5
	18	44.2	— 1	44.87	+ 1.44	43.43		18	44.2	+ 160.1	+ 18.6	+ 178.7

No. of Comparisons, (15.5.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 12\ 33.2$	$\Delta_o a = -1\ 44.42 \pm 0.18$	$\Delta_o \delta = +3\ 1.5 \pm 1.4$
$\Delta c = -\ 5\ 11.8$	$\Delta \rho = +\ 0.04$	$\Delta \rho = +\ 0.6$
	$r(a) = \pm 0^s.69$	$r(\delta) = \pm 3''.1$

Star of Comparison, p.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 9 54.60	− 1 0 7.8	1
Schjellerup	54.25	7.1	2
Lamont	54.44	5.1	1
Adopted	18 9 54.38	− 1 0 6.8	
Reduction	+ 0.49	+ 1.8	

DECEMBER 7.

Chron. Time.	Δa	Red.	$\Delta_o a$	Chron. Time.	$\Delta \delta$	Red.	$\Delta_o \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 57 27.9	− 2 56.87	− 1.10	57.97	22 57 27.9	− 171.8	− 14.2	− 186.0
23 2 37.4	− 2 59.87	+ 0.06	59.81	23 2 37.4	− 190.8	+ 0.8	− 190.0
6 59.3	− 2 60.70	+ 1.04	59.66	6 59.3	− 203.6	+ 13.4	− 190.2

No. of Comparisons, (9.3.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 2\ 21.5$	$\Delta_o \delta = -2\ 59.15 \pm 0.40$	$\Delta_o \delta = -3\ 8.7 \pm 0.9$
$\Delta c = -\ 5\ 11.9$	$\Delta \rho = +\ 0.05$	$\Delta \rho = -\ 0.6$
	$r(a) = \pm 1^s.19$	$r(\delta) = \pm 1''.6$

Star of Comparison, q.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Lamont	18 0 9.07	− 3 14 56.9	1
Schjellerup	9.38	48.8	2
Lalande	9.58	48.8	1
Adopted	18 0 9.35	− 3 14 50.8	
Reduction	+ 0.51	+ 0.9	

Collecting the results, we have the following positions of the comet. The quantities given under the head *log* $p \times \Delta$ are the logarithms of the parallactic coefficients, computed with a value of the solar parallax equal to 8″.85—the coefficient for α being expressed in seconds of time:

Date.	Washington M. T.	α	log $p \times \Delta$	δ	log $p \times \Delta$
1871.	h. m. s.	h. m. s.		° ′ ″	
November 2	7 5 6	22 36 15.83	9.0402*n*	+ 37 5 13.6	9.5130
6	7 42 7	21 56 9.70	9.0745	+ 34 33 43.7	9.8559
7	7 23 14	21 46 15.81	9.0225	+ 33 45 44.7	9.9203
8	7 51 38	21 36 6.94	9.3058	+ 32 52 3.5	0.0678
10	7 2 52	21 16 48.06	9.1754	+ 30 57 30.8	0.1248
11	7 3 3	21 7 6.54	9.2518	+ 29 54 8.8	0.1913
17	7 17 50	20 12 43.03	9.5302	+ 22 36 43.0	0.4996
25	6 26 59	19 11 16.85	9.5765	+ 11 52 15.6	0.6592
25	6 31 16	19 11 15.50	9.5824	+ 11 52 1.3	0.6613
26	6 1 27	19 4 28.14	9.5521	+ 10 32 57.6	0.6643
27	6 6 17	18 57 39.36	9.5738	+ 9 12 41.9	0.6813
29	6 22 33	18 44 25.62	9.6151	+ 6 33 43.8	0.7103
December 1	5 59 48	18 31 54.89	9.6109	+ 4 0 21.2	0.7242
2	5 57 37	18 25 48.39	9.6178	+ 2 44 41.0	0.7306
5	6 10 7	18 8 10.49	9.6472	− 0 57 2.9	0.7448
7	5 52 5	17 57 10.76	9.6482	− 3 17 59.2	0.7498

Comparing the observations with the ephemeris of S. von Glasenapp, the following residuals are found, taken in the sense of the computed minus the observed place of the comet:

Date.	In α.	In δ.
	m. s.	′ ″
November 2	− 0 57.28	+ 18 37.3
6	− 1 37.18	+ 16 51.8
7	− 1 45.50	+ 16 1.0
8	− 1 56.04	+ 15 17.6
10	− 2 18.07	+ 13 19.3
11	− 2 19.98	+ 12 0.6
17	− 2 58.55	+ 3 24.2
25	− 3 17.78	− 6 34.0
25	− 3 17.65	− 6 34.3
26	− 3 18.35	− 7 25.7
27	− 3 18.39	− 8 23.2
29	− 3 17.28	− 9 42.4
December 1	− 3 15.35	− 10 46.9
2	− 3 13.74	− 11 15.3
5	− 3 7.15	− 12 1.0
7	− 3 1.78	− 11 57.2

§ 2.

The following are the notes and remarks on the search for the comet, and its appearance after discovery:

September 8.—A careful search was made for the comet, but nothing could be seen in the place predicted by S. von Glasenapp. The sweep was extended a degree or more on each side of the computed declination, and through several degrees in right ascension.

September 21.—Looked for Encke's comet after the moon set, and made a careful search, but saw nothing.

October 9.—Looked carefully for the comet but could not see it. It appears, however, from subsequent observations, that on this night I did not extend the search far enough in declination.

October 11.—On this night a faint object was found with the comet-seeker by Paymaster H. P. Tuttle, U. S. N.; but on account of the bad seeing and the distance of the object from the predicted place, it was not certainly recognized to be the comet. An observation of the 17th proved it to be so.

October 17.—The comet was found early in the evening, as soon as the twilight was gone. It was very faint, without nucleus, and was a loose, disjointed-looking object. It appeared to be about 5′ or 6′ in diameter, with a vague and indistinct outline. No attempt was made to obtain an accurate observation for position, but an instrumental position gave the following corrections to Glasenapp's ephemeris:

$$\Delta\alpha = -38^{s} \qquad \Delta\delta = -13'$$

October 18.—The comet was examined at 10 o'clock. It was very faint and diffuse, and no trace of a nucleus could be seen. On this night the comet passed over a star of the 11.12th magnitude. The light of the star did not appear to be sensibly dimmed by its passage through the comet.

An instrumental position confirmed the corrections of the ephemeris found on the preceding night.

November 2.—Determinations of position were begun on this day. The comet had no nucleus, and was very difficult to observe.

November 6.—The comet was still very difficult to observe, but there appeared to be a little condensation at the center.

November 7.—The comet difficult to observe. It was thought to be a little elongated.

November 11.—The comet is a little brighter to-night. It appears elongated in the direction of the sun.

November 17.—The comet decidedly easier to observe. It is elongated in the direction of the sun, and has a fan-like appearance. On the following side of the comet there is a condensation, and, apparently, the beginning of a nucleus.

November 18.—Observed the spectrum of the comet in Professor Harkness's spectroscope. There were two bands visible. One of these bands was quite bright; the other a faint one.

November 25.—The comet has more condensation than hitherto. The nucleus is on the following side, the comet being elongated in the direction of the sun.

November 29.—The comet is brighter to-night and better to observe. The nucleus is on the following side of the comet.

December 1.—The nucleus is on the following side of the comet. The coma is almost wholly on the preceding side of the nucleus, and there is scarcely a trace of any on the following side. To-night the matter of the coma appears to issue from the nucleus toward the preceding side and to fall over each way, like water in the jet of a fountain. The comet was barely visible to the naked eye at 7 o'clock.

December 2.—The comet was visible to the naked eye at 6^h 30^m. The coma is mostly on the preceding side of the nucleus, and appears to flow out and fall over in curves. On the north preceding side there is a very bright part of the coma. The coma is a little on the following side of the nucleus to-night, but there it is very faint.

December 5.—The comet visible to the naked eye, and the telescope was set on it directly. It is diffuse, and the observation for position a little uncertain. The comet is so near the horizon that no very good idea can be got of its form. The coma, however, appears to be more on the following side of the nucleus than heretofore, although the most of it is still on the preceding side.

December 7.—The comet was visible to the naked eye at 6^h 10^m. In the telescope it looked fuzzy and blurred so near the horizon. The observation for position was hurried, and is, therefore, uncertain.

I first saw this comet in 1858, and have observed it at four returns. My impression is that it has been fainter during its present return than I have ever seen it before, considering its distance from the earth and sun. At this return, however, the comet approached the sun at such a time that it was quite near the earth when at a considerable distance from the sun, and in the case of this comet the formula assumed for computing the intensity of the light may not be exactly correct.

The changes of form that this comet undergoes as it approaches the sun and recedes from it have been observed at many of its returns. The general fact appears to be that there is a condensation in the matter of the comet and the formation of a nucleus as it approaches its perihelion, and an expansion of the matter and disappearance of the nucleus as the comet recedes from the sun. This appearance was observed by Struve in 1828, and his micrometrical measurements put beyond doubt the reality of the phenomenon. To an observer who sees such changes going on the question will naturally occur whether his determinations of position made at different times and when the comet has such different forms are strictly comparable. It would appear that this should be carefully considered in any discussion of the motion of the comet.

DESCRIPTION OF THE DRAWINGS.

The drawings of the comet have been placed on the page in such a way that the top of the page represents the *preceding* direction, the bottom of the page the *following* direction, the right-hand side of the page the *south*, and the left-hand side the *north*.

On October 17 the comet was very nearly circular in form, with a vague and indistinct outline. It was nearly of a uniform brightness over its whole surface, and there was no indication of a nucleus.

On November 17 the comet was elongated, the nucleus or condensed part of the comet being on the extreme following side. At this time the nucleus did not appear to be the center of action in the matter of the comet, as it appeared on December 1 and 2.

On December 1 the nucleus of the comet was a round, well-defined spot, and under a power of 132 appeared to be about 25″ or 30″ in diameter. The matter of the comet appeared to flow out from this nucleus toward the preceding side, falling over in parabolic curves about equally toward the north and south. There was very little of the coma on the following side of the nucleus. My observation extended over half an hour, and I noticed no change in the form of the comet.

On December 2 the nucleus of the comet appeared as on the 1st, and the matter of the comet appeared to flow from it in a similar manner; but on this day by far the larger portion of the coma was on the north preceding side of the nucleus. The coma appeared more on the following side of the nucleus than at any time heretofore. A line passing through the nucleus and indicating the direction of the boundary of the coma on the following side would make an angle of about 15° with a circle of declination. The curved part of the coma on the north preceding side of the nucleus was very bright.

A. HALL,
Professor of Mathematics, U. S. Navy.

Rear-Admiral B. F. SANDS, U. S. N.,
Superintendent U. S. Naval Observatory, Washington, D. C.

REPORT

OF

PROFESSOR WM. HARKNESS, U. S. N.

REPORT OF PROFESSOR HARKNESS, U. S. N.

UNITED STATES NAVAL OBSERVATORY,
Washington, February 1, 1872.

SIR: I have the honor to submit to you the following report in relation to a series of spectroscopic and polariscopic observations of Encke's Comet, made by me during the latter part of November and the beginning of December, 1871.

I.—DESCRIPTION OF INSTRUMENTS, AND METHOD OF OBSERVATION.

The *Telescope* employed was my equatorially-mounted portable achromatic refractor, of 3.91 inches clear aperture and 43.58 inches focus, made by Alvan Clark & Sons, of Cambridgeport, Massachusetts.

The *Spectroscope* was a single-prism instrument, which is fully described in my report on the total solar eclipse of August 7, 1869.[1] Its optical constants are as follows:

Small Telescope:	
Focal distance of object-glass	6.55 inches.
Clear aperture of object-glass	0.86 inch.
Diameter of field of view	5° 33′
Magnifying power	5.71 diameters.
Collimating Lens for Slit:	
Focal distance	6.52 inches.
Clear aperture	0.82 inch.
Collimating Lens for Micrometer:	
Focal distance	4.17 inches.
Clear aperture	0.82 inch.
Prism:	
Refracting angle	60° 8′
Dispersion from A to H_2	4° 5′
Minimum deviation of line D	47° 44′
Refractive index	1.613
Density	3.532

The magnifying power of this spectroscope, when used in connection with the 43½-inch telescope, is 38 diameters. The apparent length of the spectrum, as seen in the small telescope, is 23° 19′; or, in other words, it appears as a spectrum 4.12 inches long viewed at a distance of ten inches. Under favorable circumstances the line D is seen distinctly double.

[1] Washington Observations for 1867, Appendix II, pp. 27–30.

As the spectrum of Encke's Comet was very faint, the photographed micrometer-scale of the spectroscope was removed, and the micrometer, shown in Fig. 1, was inserted in its place. This consists essentially of a micrometer-screw, which moves a brass plate pierced with a hole 0.00796 of an inch in diameter. The light passing through this hole is reflected from the surface of the prism and appears in the field of view of the spectroscope-telescope as a bright disk, with an apparent diameter of 36′ 55″, which can be made to traverse the whole length of the spectrum by turning the micrometer-screw. The illumination of the disk can be adjusted to the brightness of the spectrum under observation with the greatest nicety. If it is required to be very brilliant, the direct light of a lantern may be thrown into the hole; a less degree of brightness may be secured by passing the light through a piece of ground glass; and, finally, the luminosity may be varied down to absolute invisibility by reflecting the light into the hole from the back of the observer's hand held at a suitable angle. This last plan was employed in the case of the comet. The micrometer-head is half an inch in diameter, and divided to one-tenth of a revolution, while each complete revolution of the screw moves the brass plate 0.0181 of an inch, which corresponds to an angular distance of 14′ 40″.5. The apparent motion of the luminous disk in the field of view of the spectroscope-telescope is, therefore, 1° 23′ 48″ for each revolution of the screw, or 50″.3 for the hundredth part of a revolution; and the limit of reading on the micrometer-head is thus not far from the limit of visibility in the telescope.

Fig. 1.

In using this micrometer, the readings on the line whose place was to be determined were habitually made alternately with readings on a sodium-line produced by the flame of an alcohol-lamp with a salted wick held before the object-glass of the large telescope. The measures are thus entirely differential, and there is no risk of errors having been introduced by undetected changes of zero. It has been found convenient to assume that the reading of the sodium-line is always exactly 10.00 revolutions, and to apply the difference between this assumed reading and the actual reading, at any time, as a correction to all readings on other lines made at that time. From these corrected readings the wave-lengths of the corresponding lines can at once be deduced by means of a table which was constructed from the following observations of the principal lines in the solar spectrum:

TABLE I.—*Observations of Fraunhofer's Lines to determine the value of the Spectroscope Micrometer-Screw.*

Line.	Wave-Length.	Micrometer Reading.					
		Nov. 21.	Nov. 22.	Nov. 22.	Nov. 26.	Jan. 15.	Mean.
a	718.5	. .	. .	7.00	6.99	. .	7.00
B	686.7	. .	7.63	7.60	7.56	7.61	7.60
C	656.2	8.22	. .	. .	8.21	8.25	8.23
a	627.7	. .	. .	. .	. .	8.94	8.94
D	589.2	10.00	10.00	10.00	10.00	10.00	10.00
E	526.9	12.39	. .	12.36	12.35	12.36	12.36
b	517.4	12.81	. .	. . .	12.78	12.81	12 80
F	486.1	. .	14.54	14.50	14.47	14.49	14.50
G	430.7	. .	. .	18.72	18.72	18.70	18.71
h	410.1	. .	. .	. .	20.87	20.91	20.89
H_1	396.8	. .	. .	22.57	22.55	22.57	22.56
H_2	393.3	. .	. .	23.08	23.03	23.11	23.07

The wave-lengths, which are expressed in millionths of a millimeter, are taken from Ångström's Atlas; and the observed micrometer readings have been corrected so as to make the reading of the sodium-line always 10.00 revolutions. The numbers contained in the second and eighth columns were employed to construct an interpolating curve, in the manner pointed out by Professor Pickering.[1] For that purpose a pair of rectangular co-ordinates were laid down; and then making vertical distances represent micrometer readings, and horizontal distances the squares of the reciprocals of the wave-lengths, a number of points were obtained, which necessarily lay very nearly in a straight line. A curve having been passed through these points, its abscissas were measured at the places whose ordinates corresponded to each half-revolution of the micrometer-screw, and from these measures the required wave-lengths were readily computed. For the ordinates the scale employed was one inch to each revolution of the micrometer-screw, and for the abscissas it was such that

$$\text{Log } \lambda = \frac{6.5 - \text{Log } x}{2}$$

in which λ is the wave-length, expressed in millionths of a millimeter, and x is the abscissa, expressed in inches. These scales were found quite sufficient to give the wave-lengths with as much accuracy as they can be observed in this instrument. The results are as follows:

[1] Nature, 1870, Vol. III, p. 104.

TABLE II.—*For converting Corrected Readings of the Spectroscope-Micrometer into Wave-Lengths.*

Corrected Micrometer Reading.	Wave-Length.	Difference.	Corrected Micrometer Reading.	Wave-Length.	Difference.
r.			r.		
6.0	788.9	36.8	15.5	469.8	7.5
6.5	752.1	31.5	16.0	462.7	7.1
7.0	720.6	28.4	16.5	456.1	6.6
7.5	692.2	24.9	17.0	449.8	6.3
8.0	667.3	22.7	17.5	443.9	5.9
8.5	644.6	20.2	18.0	438.2	5.7
9.0	624.4	18.4	18.5	432.8	5.4
9.5	606.0	16.8	19.0	427.7	5.1
10.0	589.2	15.6	19.5	422.7	5.0
10.5	573.6	13.8	20.0	417.8	4.9
11.0	559.8	12.5	20.5	413.3	4.5
11.5	547.3	11.9	21.0	409.1	4.2
12.0	535.4	11.2	21.5	404.9	4.2
12.5	524.2	10.6	22.0	401.1	3.8
13.0	513.6	10.0	22.5	397.4	3.7
13.5	503.6	9.3	23.0	393.8	3.6
14.0	494.3	8.7	23.5	390.5	3.3
14.5	485.6	8.3	24.0	387.3	3.2
15.0	477.3	7.5			

The diameter of the luminous disk of the micrometer is equal to 0.44 of a revolution of the screw, and it was always used as the unit when estimating distances in the spectrum. The opening of the slit was measured by holding a sodium-flame before it and comparing the width of the resulting bright band with the diameter of the micrometer-disk. When they were equal the jaws were 0.0125 of an inch apart, as may be readily computed from the constants given above.

II.—OBSERVATIONS OF ENCKE'S COMET.

Table III contains all the measures of the spectrum of the comet which were obtained at this Observatory. The micrometer readings were invariably made on the brightest part of each band.

TABLE III.

Date.	Observer.	Width of Slit.	Observed Micrometer Reading.			Mean of Observed Mic. Reading.		
			Sodium Flame.	First Band.	Second Band.	Sodium Flame.	First Band.	Second Band.
1871.		in.	r.	r.	r.	r.	r.	r.
Nov. 18	Harkness . .	0.005	13.55	. .	17.3			
			.52	. .	.3			
			.52	. .	.2	13.53	. .	17.27
18	Hall . . .	. .	13.62	. .	17.2			
			.62	. .	.25			
			.60	. .	.2	13.61	. .	17.22
25	Harkness . .	0.020	10.16	. .	13.65			
					.55			
					.53			
					.71			
			10.16	. .	.45	10.16	. .	13.58
25	Hall . . .	. .	. .	. .	13.63			
					.48			
					.40	. .	. .	13.50
26	Harkness . .	0.020	10.38	. .	13.95			
			.40	. .	.68			
			.42	. .	.80	10.40	. .	13.81
27	Harkness . .	0.022	10.42	. .	13.65			
			.41	. .	.67			
			.42	. .	.60			
			.43	. .	.62			
			.44	. .	.62	10.43	. .	13.63
29	Harkness . .	0.020	10.51	. .	13.68			
			.50	. .	.68			
			.50	. .	.70			
			.50	. .	.77			
			.50	12.3	.69	10.50	12.3	13.70
Dec. 1	Harkness . .	0.017	10.51	. .	13.57			
			.51	. .	.70			
			.51	11.95	.70			
			.49	.99	.62			
			.48	.80	.64	10.50	11.91	13.65
2	Harkness . .	0.020	10.50	. .	13.60			
			.50	. .	.55			
			.50	11.88	.68			
			.45	.95	.52			
			.49	.92	.61	10.49	11.92	13.59

The following are the notes which accompanied these observations:

November 18, 1871.—A fine moonless evening. Found the spectrum of Encke's Comet to consist of two bright bands, in each of which the light was most intense in the middle of its breadth, and shaded off toward the edges. The more refrangible band was certainly three or four times brighter than the other, and was about equal in breadth to the diameter of the micrometer-disk. The fainter band was, perhaps, a little narrower. No continuous spectrum was visible.

November 25.—Moon nearly full and atmosphere a little hazy; consequently, the spectrum of the comet was very difficult to observe, the bright band being excessively faint, and the other one not sufficiently visible to enable me to estimate its position satisfactorily. As the slit was gradually closed the bands slowly faded away without getting perceptibly narrower. In the large equatorial of $9\frac{1}{2}$ inches aperture the comet appeared as a faint nebulous cloud, about four or five minutes in diameter, with a slight condensation at its center, but no trace of a nucleus.

November 26.—Moon nearly full, and the sky, in addition to being quite hazy, was covered with drifting clouds, between which the observations were made. The comet was very faint indeed, and its spectrum so dim that the less refrangible band was only visible by glimpses. Both last night and to-night I have seen a faint continuous spectrum, but it appears to be due to the moonlight, for it was found in all parts of the sky to which the telescope was pointed.

November 27.—A fine, clear evening, with the full moon about ten degrees high. The distance of the less refrangible band of the spectrum from the other was estimated to be two and a half times the diameter of the micrometer-disk. This makes its reading $13^r.63 - 2.5 \times 0^r.44 = 12^r.53$. The more refrangible band is certainly three times as bright as the other. The faint continuous spectrum from the moonlight in the sky is still visible.

November 29.—The evening was very clear and the observations were finished before the moon rose. The breadth of each of the bands composing the spectrum was the same, and about equal to the diameter of the micrometer-disk. The more refrangible band was certainly three or four times brighter than the other. No trace of a continuous spectrum was detected.

December 1.—A fine, clear, moonless evening. The comet was brighter than ever before, and just barely visible to the naked eye. In the $9\frac{1}{2}$-inch equatorial it showed considerable condensation, which, however, was not central, but mostly on the side farthest from the sun. Upon a careful examination of the spectrum in an absolutely dark field, I discovered a third band, which I estimated to be more refrangible than either of the other two by about twice the interval between them. This makes its micrometer reading $13^r.65 + 2 \times 1^r.74 = 17^r.1$. If we designate the least refrangible as the first, the next as the second, and the most refrangible as the third; then calling the brightness of the third band unity, that of the second was approximately sixteen, and that of the first four. The brightest part of the first and second bands was altogether on the less refrangible side of their centers, and was about equal in breadth to the diameter of the micrometer-disk; while the total breadth of each band was about twice the diameter of that disk. The third band was of nearly uniform

brightness throughout its whole breadth, which was also about twice the diameter of the micrometer-disk. Upon narrowing the opening of the slit, the bands did not become lines, but gradually faded out. I looked very carefully for a continuous spectrum, but am in doubt whether or not one existed.

I examined the comet closely for polarization, employing a comet-seeker of four inches aperture and a double-image prism, but no change in the relative brightness of the two images was perceptible on rotating the prism.

December 2.—A fine, clear, moonless evening, but the comet had an altitude of only twenty degrees and seemed scarcely so bright as it was yesterday. The third band in its spectrum was barely visible, and I did not even attempt to estimate its position. During all my observations of this comet I have fancied, at times, that it gave a faint continuous spectrum, but, upon looking carefully for it, I never could feel certain of its existence.

Table IV contains the final numerical results of the observations on the spectrum of the comet. The corrected micrometer readings were obtained by subtracting from the observed, or estimated, readings the quantity necessary to reduce the mean of the readings on the sodium-flame to 10.00 revolutions, as already explained. The wave-lengths are expressed in millionths of a millimeter, and have been taken from Table II, with the argument "Corrected micrometer reading."

TABLE IV.

Date.	Corrected Micrometer Reading.			Wave-Length.		
	First Band.	Second Band.	Third Band.	First Band.	Second Band.	Third Band.
1871.	r.	r.	r.			
Nov. 18	. .	13.68	. .	. .	500.2	
25	. .	.38	. .	. .	505.9	
26	. .	.41	. .	. .	505.4	
27	12.10	.20	. .	533.	509.6	
29	11.8	.20	. .	540.	509.6	
Dec. 1	.41	.15	16.6	549.5	510.6	455.
2	.43	.10	. .	549.0	511.6	

Perhaps the most remarkable feature about this series of observations is the continual increase in the wave-length of the light emitted by the brightest part of the second band of the spectrum. If the amount of the change had been small it might have been attributed to accidental errors in pointing the micrometer, but it is much too great to admit of such an explanation. Zöllner has shown that "the ratio of brightness of two adjacent places in the spectrum (of a gas) may be reversed by alterations of temperature, and a minimum appear in the place of a former maximum."[1] This is probably the key to the change in question. The comet was approaching the sun.

[1] Zöllner on the Influence of Density and Temperature on the Spectra of Incandescent Gases. L. E. & D. Phil. Mag., 1871, Vol. 41, p. 199.

Its temperature must, therefore, have been increasing; and, in consequence of that increase, the character of its light was slightly modified. So far as I am aware, this is the first time that such a phenomenon has been observed in a celestial body.

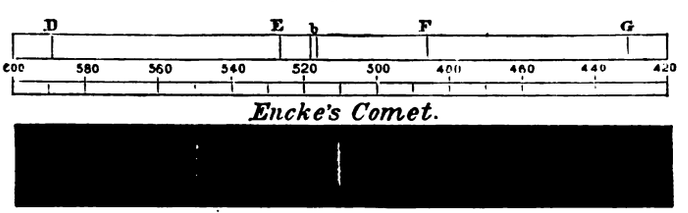
The comet was decidedly best seen on the evening of December 1; and, by the joint use of the micrometer readings and notes made on that occasion, Table V has been constructed. It gives the numerical data for the most important features of the spectrum, but it must be remembered that the positions of the *edges* of the bands were only estimated.

TABLE V.

	Encke's Comet.			Comet II, 1868.
	Relative Brightness.	Corrected Micrometer Reading.	Wave-Length.	Wave-Length.
		r.		
First Band	4			
Less Refrangible Edge	.	11.19	555.	562.
Brightest Part	.	11.41	549.5	
More Refrangible Edge	.	12.07	533.	542.
Second Band	16			
Less Refrangible Edge	.	12.93	515.	516.
Brightest Part	.	13.15	510.6	
More Refrangible Edge	.	13.81	497.	499.
Third Band	1			
Less Refrangible Edge	.	16.16	461.	471.
Middle	.	16.6	455.	
More Refrangible Edge	.	17.04	449	459.

Fig. 2 conveys a very good idea of the general appearance of the spectrum, and, in addition, shows the position of the bright bands relatively to some of the most conspicuous lines in the solar spectrum.

Fig. 2.



Mr. Huggins has published a careful description of the spectrum of Comet II, 1868,[1] but, as he has not furnished the means of converting his micrometer readings

[1] Further Observations on the Spectra of some of the Stars and Nebulæ, with an Attempt to determine therefrom whether these Bodies are moving toward or from the Earth; also Observations on the Spectra of the Sun and of Comet II, 1868. Phil. Trans., 1868, p. 555.

into any known scale, in order to obtain the wave-lengths it is necessary to have recourse to the plate accompanying his paper. In addition to the spectrum of the comet, this exhibits also some of the principal lines of the solar spectrum, and by interpolating between them the numbers contained in the last column of Table V have been deduced. A comparison of these numbers with those contained in the preceding column of the same table will show that the wave-lengths of the spectrum of Encke's Comet are so nearly identical with those of Comet II, 1868, as to render it almost certain that the differences between them are entirely due to the far from accurate methods employed in their determination. When, in addition to this, the great similarity in the general appearance of the two spectra is considered, it seems impossible to avoid concluding that they are absolutely identical. Hence it follows that the physical constitutions of the two comets are also identical; and, as Mr. Huggins has shown that Comet II, 1868, is probably composed mainly of incandescent carbon in a gaseous state, we infer that Encke's comet is also composed of the same substance. At all events, my observations show that it gives a carbon-spectrum exactly like that obtained from an electric spark taken in olefiant gas.

III.—DENSITY OF ENCKE'S SUPPOSED RESISTING MEDIUM IN SPACE.

As it was from a consideration of the motion of this comet that Encke was led in 1819 to propound his famous theory of a resisting medium in space, perhaps it may not be inappropriate to incorporate in this report an attempt to determine the density of that medium, but in so doing I do not wish to be understood as expressing any opinion with regard to the truth or falsity of Encke's hypothesis. The problem to be considered may be stated as follows:

Supposing all space to be pervaded by a resisting medium of uniform density, what must that density be in order to account for the observed retardation of Encke's Comet?

1.—*Investigation of a General Expression for the Mass of a Comet.*

If we assume the sun's horizontal equatorial parallax to be 8″.9, and adopt Captain Clarke's elements of the figure of the earth,[1] namely,

Equatorial semi-diameter	3963.057 miles,
Polar semi-diameter	3949.760 miles,

we shall have, for the distance of the earth from the sun,

$$\frac{3963.06}{\sin 8''.9} = 91\ 847\ 000 \text{ miles.}$$

[1] Phil. Trans., 1856, p. 626.

The following are the principal results which have hitherto been obtained for the average specific gravity of the earth, water being taken as unity:

Schehallien experiment[1]	5.0
Cavendish's experiment, reduced by Baily[2]	5.45
Reich, by Cavendish's method	5.44
Baily, by Cavendish's method[3]	5.67
Airy, by his Harton Colliery experiment[4]	6.565
Captain Clarke, from deflections of the plumb-line at Arthur's Seat[5]	5.316

The mean of all is 5.57, which I shall use as the best value at present attainable.

The following notation will be employed:

Δ = mean distance of the earth from the sun = 91 847 000 English miles.

ρ'' = distance of the comet from the earth, expressed in terms of the earth's mean radius-vector.

s = apparent semi-diameter of the comet as seen from the earth, expressed in arc.

m = mass of comet.

σ = specific gravity of the gas composing the comet, taken at its actual pressure and temperature, which are supposed to be t degrees F. and p inches of mercury.

σ_0 = specific gravity of the gas composing the comet, taken at a temperature of 60° F., and a pressure of 30 inches of mercury at 60° F., the specific gravity of air being considered as unity.

π = ratio of the circumference of a circle to its diameter = 3.14159.

w = weight of one cubic foot of air at a temperature of 60° F. and a pressure of 30 inches of mercury = 534.55 grains.

f = number of feet in one English mile = 5280.

γ = number of grains in one avoirdupois pound = 7000.

β = number of pounds in one ton = 2000.

If $V_{(p,\,t)}$ is the volume, and σ the specific gravity, of a certain quantity of gas at the pressure p and temperature t, and $V_{(p',\,t')}$ its volume, and σ' its specific gravity, at the pressure p' and temperature t', then

$$\sigma\, V_{(p,\,t)} = \sigma'\, V_{(p',\,t')}$$

and, also,

$$V_{(p,\,t)} = V_{(p',\,t')}\,\frac{(1+\alpha t)\,p'}{(1+\alpha t')\,p}$$

where α is the coefficient of expansion, which is nearly the same for all gases. Hence

$$\sigma = \sigma'\,\frac{(1+\alpha t')\,p}{(1+\alpha t)\,p'}$$

[1] Abridg. Phil. Trans., 1776-80, Vol. XIV, p. 420.
[2] Mem. Roy. Ast. Soc., Vol. XIV, p. 91.
[3] *Ibid.*, p. ccxlvii.
[4] Phil. Trans., 1856, p. 342.
[5] *Ibid.*, p. 606.

Employing the Fahrenheit scale of temperatures, for which $\alpha = \frac{1}{459}$, and putting $p' =$ 30 inches, and $t' = 60°$, this becomes

$$\sigma = \sigma_0 \frac{519\,p}{(459 + t)\,30} \qquad (1)$$

by means of which the specific gravity of a comet may be computed, provided the gas of which it is composed, and also its pressure and temperature, can be discovered.

If the distance of a comet from the sun is $\Delta\rho''$ miles, its radius will be $\Delta\rho''$ sin s miles, and its volume will be $\frac{4}{3}\pi(\Delta\rho''\sin s)^3$ cubic miles. One cubic foot of its substance will weigh σw grains, and one cubic mile will weigh $\sigma w f^3 \div \beta\gamma$ tons. The total mass of the comet will, therefore, be

$$\frac{\frac{4}{3}\pi w f^3 \Delta^3}{\beta\gamma}\,\sigma(\rho''\sin s)^3 = 1\,824 \times 10^{23}\,\sigma(\rho''\sin s)^3 \text{ tons.} \qquad (2)$$

Taking the mean radius of the earth as 3956.4 miles, its volume is $\frac{4}{3}\pi\,3956.4^3$ cubic miles. As already shown, its density is 5.57 times that of water, which weighs 62.50 pounds per cubic foot. Hence, the total weight of the earth is $\frac{4}{3}\pi \times 5.57 \times 62.50 \times 3956.4^3 f^3 \div \beta = 6\,647 \times 10^{18}$ tons; and the mass of the comet, expressed in terms of the earth's mass as unity, will be

$$\frac{1\,824 \times 10^{23}}{6\,647 \times 10^{18}}\,\sigma(\rho''\sin s)^3 = 2\,744 \times 10^6\,\sigma(\rho''\sin s)^3 \qquad (3)$$

Lead weighs 709.5 pounds per cubic foot; hence, a sphere of that metal having a radius of L miles will weigh $\frac{4}{3}\pi \times 709.5\, f^3 L^3 \div \beta = 2\,187 \times 10^{18} L^3$ tons. Equating this with the right-hand member of equation (2) we find that the weight of the comet will be equivalent to that of a sphere of lead having a radius of

$$4\,369\,000\,\sqrt[3]{\sigma}\,(\rho''\sin s) \text{ miles.} \qquad (4)$$

2.—*Investigation of a General Expression for the Amount of Free Force developed by the shortening from a to a' of the Semi-axis Major of a Body moving about the Sun in a Closed Orbit.*

In addition to the notation already given, the following will be required:

V = velocity of the body in its orbit at the time its radius-vector is ρ'.

a, a' = semi-axes major at the times T and T', expressed in terms of the earth's mean radius-vector as unity.

ρ, ρ' = radii-vectores at the times T and T'.

v = true anomaly.

k = a constant peculiar to the solar system. Log $k = 8.23558$.

g = mean force of gravity at the surface of the earth, expressed in terms of the velocity which a body falling freely would acquire at the end of the first second = 32.180 feet.

If a body whose mass is insignificant compared with that of the sun move about the latter in a closed orbit, its velocity, at any time, will be given by the equation

$$V^2 = k^2\left(\frac{2}{\rho'} - \frac{1}{a}\right)$$

As the units in this expression are the earth's mean radius-vector and the mean solar day, in order to obtain V in feet per second the right-hand member of the equation must be multiplied by the square of the number of feet in the earth's mean distance from the sun, and divided by the square of the number of seconds in a day. We, therefore, have

$$V^2 = \left(\frac{2}{\rho'} - \frac{1}{a}\right)\frac{k^2\Delta^2 f^2}{86\,400^2} = 9\,322 \times 10^6\left(\frac{2}{\rho'} - \frac{1}{a}\right) \qquad (5)$$

Imagining the sun's whole mass to be concentrated at its center, we may suppose the orbit of the body to become more and more eccentric, until finally it approaches indefinitely near a straight line, having the foci at its two extremities. Under such circumstances it is evident that when the body attained its aphelion it would be, for an instant, in a state of absolute rest, and then it would begin its return to the sun, which it would approach with ever-increasing velocity until it arrived at perihelion, when, for an instant, its velocity would be infinite; and then it would again recede, with diminishing velocity, to its aphelion, and thus continue to oscillate back and forth for evermore. If the aphelion distance is designated by ρ, the mean distance will evidently be $\frac{1}{2}\rho$, and we shall have

$$V^2 = 9\,322 \times 10^6\left(\frac{2}{\rho'} - \frac{2}{\rho}\right)$$

A moment's consideration will show that this expression applies also to the case of a body situated at a distance ρ from the sun, and falling toward it, from a state of rest, through the distance $\rho - \rho'$. But the work done in imparting a velocity of V feet per second to a body having a mass of m pounds is

$$\frac{m\,V^2}{2\,g} \text{ foot-pounds.}$$

Hence, the amount of energy which the sun would impart to a body while falling through the space of $\rho - \rho'$ is

$$1\,448 \times 10^5\left(\frac{2}{\rho'} - \frac{2}{\rho}\right) m \text{ foot-pounds.} \qquad (6)$$

Resuming equation (5), it is evident that if at the epoch T we suppose the body to have had the true anomaly v, the semi-axis major a, and the radius-vector ρ, its velocity in its orbit was

$$\left\{9\,322 \times 10^6\left(\frac{2}{\rho} - \frac{1}{a}\right)\right\}^{\frac{1}{2}} \text{ feet per second,}$$

and the work required to produce this velocity was

$$1\,448 \times 10^5 \left(\frac{2}{\rho} - \frac{1}{a}\right) m \text{ foot-pounds.}$$

Again, if at the epoch T', the eccentricity of the orbit and the true anomaly being the same as before, we suppose the body to have had the semi-axis major a', and the radius-vector ρ', the work required to produce the velocity which it then had in its orbit was

$$1\,448 \times 10^5 \left(\frac{2}{\rho'} - \frac{1}{a'}\right) m \text{ foot-pounds.}$$

Hence, while the radius-vector shortened from ρ to ρ', a force of

$$1\,448 \times 10^5 \left(\frac{2}{\rho'} - \frac{2}{\rho} + \frac{1}{a} - \frac{1}{a'}\right) m \text{ foot-pounds}$$

was expended in increasing the velocity of the body's motion. Subtracting this from the total amount of energy imparted by the sun's attraction during the same time, as given by equation (6), there remains a free force of

$$1\,448 \times 10^5 \left(\frac{1}{a'} - \frac{1}{a}\right) m \text{ foot-pounds,} \qquad (7)$$

which may have been applied to overcoming the resistance of a medium pervading space.

3.—*Investigation of a General Expression for the Density of a Medium, in Terms of the Resistance experienced by a Body of Known Size moving through it with known Velocity.*

It is a very long step from the speed of a cannon-ball to that of a planet or comet; and yet, as the highest velocities with which we have hitherto been able to experiment are furnished by projectiles, in an investigation like the present we have no recourse except to argue from the resistance which they experience to that which may be experienced by bodies moving about the sun.

The law connecting the velocity of a projectile with the resistance offered to it by the air does not seem to be well determined, but perhaps the most satisfactory formula is that of Mr. J. A. Longridge,[1] who gives

$$\text{Log } R = 4.5 \log V - 13.0155$$

where R is the resistance per square inch of sectional area, expressed in avoirdupois

[1] Proceedings Royal Society, 1868, Vol. XVI, p. 265; or, L. E. & D. Phil. Mag., 1868, Vol. XXXV, p. 305.

pounds, and V is the velocity in feet per second. If R is taken in tons per square mile, the formula will become

$$\text{Log } R = 4.5 \log V - 6.7129$$

that is,

$$R = \frac{V^{4.5}}{5\,163\,000}$$

which corresponds to an atmospheric pressure of about thirty inches of mercury. If the sectional area of the projectile is α square miles, and the resistance is supposed to vary as the pressure, then, for a pressure of p inches, the total resistance will be

$$R = \frac{\alpha p V^{4.5}}{154\,900\,000}$$

Hence, if we know the velocity of a body, its sectional area, and the resistance with which it meets, we can compute approximately the density of the medium through which it is moving by means of the formula

$$p = \frac{154\,900\,000\, R}{\alpha V^{4.5}} \qquad (8)$$

4.—*Investigation of a General Expression for the Density of the Supposed Resisting Medium in Space.*

If a body moving about the sun in a closed orbit passes over a distance of F feet between the epochs T and T', and if, during that period, its semi-major axis is shortened from a to a', then it results from equation (7) that it must have experienced a constant resistance equal to

$$\frac{1\,448 \times 10^5}{F}\left(\frac{1}{a'} - \frac{1}{a}\right) m$$

Substituting for m its value from equation (2), this becomes

$$\frac{1\,448 \times 10^5}{F} \times \frac{\frac{4}{3} w f^3}{\beta \gamma} \sigma \pi (\Delta \rho'' \sin s)^3 \left(\frac{1}{a'} - \frac{1}{a}\right)$$

which is the value of R in equation (8). α, in the same equation, is equal to $\pi(\Delta \rho'' \sin s)^2$. Substituting these values of R and α, and reducing, equation (8) becomes

$$p = \frac{1\,544 \times 10^{28} \rho'' \sin s \left(\frac{1}{a'} - \frac{1}{a}\right) \sigma}{F V^{4.5}} \qquad (9)$$

which is the formula required.

5.—*Application of the Equations found in Paragraphs* 1, 2, 3, *and* 4 *to the case of Encke's Comet.*

As already explained, the spectroscope shows that Encke's Comet is mainly, perhaps entirely, composed of carbon in a gaseous state, and that its spectrum is identical with that of olefiant gas, except that it lacks the hydrogen lines. But, owing to the failure of physicists in obtaining carbon vapor, no determination of its specific gravity exists, and, as a rough approximation, I shall therefore employ the specific gravity of olefiant gas, which, being a hydro-carbon, is probably lighter than the material composing the comet.

It is next necessary to have an estimate of the mean pressure and temperature of the gas which forms the comet. In judging of the first of these elements, there is scarcely anything to guide us, and I have somewhat arbitrarily assumed that it is not greater than 0.12 of an inch, nor less than 0.004 of an inch. As to the second element, Kirchhoff has shown that the temperature of a gaseous body, yielding a discontinuous spectrum of given wave-lengths, cannot be less than that of a perfectly black body whose spectrum is of the same brightness at the same wave-lengths, and this renders it certain that the temperature of the comet was not much less than 1 000° F. It is equally sure that it was not greater than that of the sun, which we will now investigate.

Secchi, from observations on radiation, estimates the temperature of the sun at 10 000 000° C.;[1] Ericsson, from observations of the same kind, made by means of a peculiar apparatus of his own, estimates it at 4 036 000° F. = 2 243 000 C.;[2] Zöllner, from his theory of the cause of the red prominences, estimates it at not less than 68 400° C.;[3] Lane, from the recorded effects of Parker's great burning-lens, estimates it at 55 450° F. = 30 788° C.;[4] and Spœrer estimates it at 27 000° C. Although these estimates differ widely from each other, they all seem to me extremely high, and probably vastly in excess of any temperature that ever existed in the universe. In the year 1838 Pouillet estimated the temperature of the sun as somewhere between 1 461° and 1 761° C.[5] Last January M. E. Vicaire, using Secchi's data, and reducing them by means of Dulong and Petit's formula for the intensity of radiation, found that the resulting temperature of the sun was 1 398° C., and, from an extended discussion of the subject, he concluded that the temperature of the solar surface is entirely comparable with that of terrestrial flames.[6] His paper was presented to the French Academy, and, after its reading, a number of the *savants* present expressed their substantial concurrence in his views, their estimates of the probable solar temperature all lying between 2 500° and 3 000° C.[7]

If R represents the total radiation in vacuo from a body whose temperature is $t + \theta$, situated in an inclosure whose temperature is θ, then, from a series of experiments, in which the value of $t + \theta$ ranged between 60° and 240° C., and that of θ was varied from 0° to 80° C., Dulong and Petit found that[8]

$$R = m\,\alpha^{\theta}(\alpha^{t} - 1)$$

[1] Le Soleil, p. 271.
[2] Nature, 1871, Vol. IV, p. 452.
[3] L. E. & D. Phil. Mag., 1870, Vol. XL, p. 323.
[4] Amer. Jour. of Science, 1870, Vol. L, p. 68.
[5] Comptes Rendus, 1838, T. VII, p. 35.
[6] *Ibid.*, 1872, T. LXXIV, pp. 33 and 34.
[7] *Ibid.*, pp. 35–36 and 152.
[8] Annales de Chimie et de Physique, 1817, T. VII, p. 252.

where α is a constant the numerical value of which for the centigrade scale of temperatures is 1.0077, and m is an unknown constant depending upon the unit of heat adopted, the size of the radiating body, the substance of which it is composed, and the time during which the radiation takes place. Assuming the truth of this equation, adopting as the unit of heat the quantity required to raise 1 000 grains of water 1° C., and making R the amount of radiant heat which emanates in one minute from a surface one foot square, Mr. Hopkins determined experimentally the approximate value of m for several different substances, as follows:[1]

Glass	9.566
Dry chalk	8.613
Dry new red sandstone	8.377
Sandstone, (building-stone)	8.882
Polished limestone	9.106
Unpolished limestone, (same block)	12.808

It is more convenient to take as the unit of heat the quantity required to raise one gram of water from 0° to 1° C., and to make R the radiation in one minute from a surface one centimeter square. As 1 000 grains are equal to 64.82 grams, and one square foot is equal to 929.0 square centimeters, in order to make the change it is only necessary to multiply the values given above by $\frac{64.82}{929.0} = 0.06978$. We then have

Glass	0.6674
Dry chalk	.6009
Dry new red sandstone	.5845
Sandstone, (building-stone)	.6197
Polished limestone	.6353
Unpolished limestone, (same block)	.8937

If r represents the intensity of solar radiation on a surface one centimeter square, situated at a distance of Δ centimeters from the sun, then the total solar radiation will be $4\pi\Delta^2 r$; and if the apparent angular semi-diameter of the sun as seen from the distance Δ is s, its radius will be $\Delta \sin s$, and its surface will be $4\pi\Delta^2 \sin^2 s$. Hence, the radiation at the solar surface from an area one centimeter square will be $\frac{r}{\sin^2 s}$, and, in accordance with Dulong and Petit's law, we shall have

$$\frac{r}{\sin^2 s} = m\,\alpha^\theta(\alpha^t - 1)$$

which at once gives

$$t + \theta = \frac{\log\left(\alpha^\theta + \frac{r}{m \sin^2 s}\right)}{\log \alpha} \qquad (10)$$

If, in this formula, while m and α retain the same signification as before, we make θ represent the temperature of space beyond the limit of the earth's atmosphere, r the

[1] Phil. Trans., 1860, p. 407.

intensity of solar radiation at the earth's surface, corrected for atmospheric absorption, and s the sun's semi-diameter at the earth's mean distance, then $t + \theta$ will be the temperature of the sun.[1]

Probably the best determinations of the temperature of space are those of Pouillet and Sir J. F. W. Herschel. The former gives — 142° C.,[2] and the latter — 239° F. = — 151° C.[3] The mean is — 146° C., which I adopt as the value of θ. Taking as the unit of heat the quantity required to raise one gram of water from 0° to 1° C., Pouillet found, from observations made at Paris, that if the atmosphere did not exist, the sun's rays, falling vertically on a surface one centimeter square, would be capable of imparting to it 1.7633 units of heat per minute.[4] Sir J. F. W. Herschel found, from observations made at the Cape of Good Hope, that when the sun was within 12° of the zenith its rays, after passing through the earth's atmosphere, were capable of melting 0.1914 of a millimeter of ice per minute.[5] Employing the value used by him for the latent heat of ice, viz, 135°.5 F. = 75°.28 C., and, taking the atmospheric absorption within 12° of the zenith to be 0.25, as given by Pouillet, this becomes 1.801 units of heat for each square centimeter of surface. The mean between this result and that of Pouillet is 1.782 units, which I adopt as the value of r. The value of s is 16′ 2″. Mr. Hopkins's experiments seem to indicate that for the substances composing the earth's crust the value of m is about 0.67, but, as we have no knowledge of its value for the sun, I shall assume that it lies somewhere between 0.1 and unity.

Collecting our results, we have

$\theta = -146°$ C. $\quad r = 1.782 \quad s = 16' \ 2''$
m lies somewhere between 0.1 and 1.0

Substituting these values in equation (10) we obtain

m	$t + \theta$	m	$t + \theta$
0.1	1775° C.	0.6	1542° C.
0.2	1685	0.7	1522
0.3	1632	0.8	1504
0.4	1594	0.9	1489
0.5	1565	1.0	1475

In order to make $t + \theta$ as great as 2 000° C. we must assume m as small as 0.018; and, if Dulong and Petit's formula holds for such elevated temperatures, it does not seem possible that the sun can be hotter than the oxyhydrogen flame, the temperature of which, according to Bunsen, is about 2 844° C.[6]

[1] This equation is essentially the same as that given by Pouillet in the Comptes Rendus, 1838, T. VII, p. 35; but, as that work is not generally accessible in this country, and as the above was written before I had seen Pouillet's memoir, it is allowed to remain in this report.

[2] Comptes Rendus, 1838, T. VII, p. 61.

[3] Encyclopædia Britannica, 8th edition, Art. Meteorology, par. 36.

[4] Comptes Rendus, 1838, T. VII, p. 31.

[5] Results of Astronomical Observations made at the Cape of Good Hope, by Sir J. F. W. Herschel, p. 446.

[6] Poggendorf's Annalen, Vol. CXXXI, p. 172.

Returning to the consideration of the comet, in view of what has just been said, it is very unlikely that its temperature approached 5 000° F.; but, in order to be on the safe side, I will assume that t was not less than 1 000°, nor greater than 20 000° F. Collecting our results, we have $\sigma_0 = 0.9784$, p comprised between the limits 0.004 and 0.12 of an inch of mercury, and t comprised between the limits 1 000° and 20 000° F. Substituting these values in equation (1), we obtain the values of $\log \sigma$ given in

TABLE VI.

NOTE.—In using these logarithms their characteristics must be diminished by 10.

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	5.6666	5.0935	4.8111	4.5197
.04	6.6666	6.0935	5.8111	5.5197
.08	6.9676	6.3946	6.1122	5.8208
.12	7.1437	6.5706	6.2883	5.9969

On the 25th of November the apparent diameter of the comet was from four to five minutes. In order to be on the safe side I will take $s = 2'$. For that date Mr. S. von Glasenapp's ephemeris[1] gives $\log \rho'' = 9.5170$. Hence, $\log (\rho'' \sin s) = 6.2818 - 10$. Substituting that value in equation (3), it becomes $0.01922\ \sigma$, by means of which Table VII has been computed.

TABLE VII.—*Mass of Encke's Comet in terms of the Earth's Mass as Unity.*

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	0.000 000 892	0.000 000 238	0.000 000 124	0.000 000 064
.04	.000 008 920	.000 002 384	.000 001 244	.000 000 636
.08	.000 017 84	.000 004 768	.000 002 489	.000 001 273
.12	.000 026 76	.000 007 152	.000 003 733	.000 001 909

Substituting in equation (4) the value of ρ'' sin s, given above, it becomes $836.0\sqrt[3]{\sigma}$, and the formula for the diameter in miles of a sphere of lead having the same mass as the comet is $1\ 672\sqrt[3]{\sigma}$, by means of which Table VIII has been computed.

[1] Astronomische Nachrichten, No. 1854, Band 78, p. 90.

TABLE VIII.—*Diameter in Miles of a Sphere of Lead whose Mass is equal to that of Encke's Comet.*

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	60.1	38.70	31.16	24.91
.04	129.4	83.37	67.12	53.67
.08	163.1	105.05	84.58	67.63
.12	186.6	120.26	96.83	77.42

The extreme levity of comets has been so strongly insisted upon that the numbers in Tables VII and VIII seem, at first sight, surprisingly large, and perhaps many persons will feel inclined to doubt their accuracy when they remember that on August 23, 1779, Lexell's comet passed nearer to the planet Jupiter than its fourth satellite, and, although none of the satellites suffered any perceptible perturbation, the orbit of the comet was completely changed. However, that fact is entirely consistent with the numbers given in the Tables, for the mass of Jupiter's smallest satellite is 0.005 868, while the largest mass deduced for the comet is only 0.000 027; or, in other words, the mass of the satellite is at least 219 times greater than that of the comet, and no perturbations visible from the earth were to be expected.

The mean density of the earth is 2.037 times less than that of lead. Hence, multiplying the numbers in Table VIII by $\sqrt[3]{2.037} = 1.268$, we find that the mass of the comet was approximately equal to that of a sphere having the mean density of the earth and a diameter somewhere between 32 and 237 miles. Mr. Stone has shown that the diameters of the first seventy-one asteroids are probably all included between the limits 17 and 214 miles.[1] Their density is unknown, but it is not likely to exceed that of the earth. We are therefore led to conclude that *the mass of Encke's Comet is not less than that of an asteroid*, and this conclusion cannot be modified by any probable change in the adopted value of σ_0; for, even if we suppose the comet to consist of pure hydrogen, existing at the pressures and temperatures given in Table VIII, its mass must have been equal to that of a sphere having the mean density of the earth and a diameter somewhere between 13 and 98 miles. Few persons realize that gases are heavy, and yet one cubic mile of atmospheric air weighs 5 621 000 tons, and one cubic mile of pure hydrogen, the lightest known substance, weighs no less than 389 000 tons.

In order to obtain numerical values of a, a', F, and $V^{4.5}$, in equation (9), it is necessary to have the elements of the orbit of Encke's Comet, at two epochs, as widely

[1] Approximate Relative Dimensions of Seventy-one of the Asteroids, by E. J. Stone. Monthly Notices; Roy. Ast. Soc., 1867, Vol. XXVII, p. 302.

separated as possible. For the first epoch, I will take the perihelion passage of 1819, January 27.2934, Berlin mean time, when, according to Encke,[1]

$$e = 0.84858$$
$$\log q = 9.52538$$
$$\log a = 0.34520$$

and, for the second epoch, I will take the perihelion passage of 1871, December 28.6297, Berlin mean time, when, according to Mr. S. von Glasenapp,[2]

$$e' = 0.84936$$
$$\log q' = 9.52229$$
$$\log a' = 0.34433$$

Taking the mean of these two sets of elements, we obtain

$$e_0 = 0.84897$$
$$\log q_0 = 9.52384$$

Whence, by means of the formulæ,

$$a = \frac{q}{1 - e}$$

$$b = a(1 - e^2)^{\frac{1}{2}}$$

$$\mu = \frac{3548''.2}{a^{\frac{3}{2}}}$$

$$\text{Periodic Time} = \frac{360^\circ}{\mu}$$

we obtain

$$\log a_0 = 0.34478$$
$$\log b_0 = 0.06778$$
$$\mu_0 = 1078''.55$$
$$\text{Periodic Time} = 1201.6 \text{ days} = 3.2899 \text{ years.}$$

During the interval between the adopted epochs, the mean length of the circumference of the comet's orbit was approximately

$$2\pi a_0(1 - 0.25\, e_0^2 - 0.046\,875\, e_0^4 - 0.019\,532\, e_0^6)$$

the numerical value of which is 10.95 times the earth's mean radius-vector. But, between January, 1819, and December, 1871, the comet made sixteen complete revo-

[1] Olbers Abhandlung, Encke's edition, p. 184. [2] Astronomische Nachrichten, No. 1854, Band 78, p. 90.

lutions, and therefore traveled $16 \times 10.95 = 175.2$ mean terrestrial radii-vectores, or $8\,498 \times 10^{10}$ feet, which is the value of F in equation (9).

Let

$T =$ time of perihelion passage,
$e =$ eccentricity,
$q =$ perihelion distance,
$a =$ semi-major axis,
$b =$ semi-minor axis,
$\mu =$ mean daily motion,
$M =$ mean anomaly at the time t,
$E =$ eccentric anomaly at the time t,
$v =$ true anomaly at the time t,
$\rho =$ radius-vector at the time t,
$V =$ velocity in the orbit, in feet per second, at the time t;

then we have the well-known formulæ

$$\mu(t - T) = M$$

$$M = E - e \sin E$$

$$\tan \tfrac{1}{2} v = \sqrt{\frac{1 + e}{1 - e}} \tan \tfrac{1}{2} E$$

$$\rho = a(1 - e)\left(\frac{\cos \tfrac{1}{2} E}{\cos \tfrac{1}{2} v}\right)^2$$

and also equation (5), which is

$$V^2 = 9\,322 \times 10^6 \left(\frac{2}{\rho} - \frac{1}{a}\right)$$

by means of which the logarithmic values of $V^{4.5}$ have been computed from the elements given above, as follows:

	d.		d.	d.	d.	d.	d.	d.	d.	d.
$t - T$	0.00	20.03	40.05	60.08	120.16	180.24	240.32	360.48	480.64	600.80
Log $(t - T)$	. . .	1.30168	1.60260	1.77873	2.07976	2.25585	2.38079	2.55688	2.68182	2.77873
Log μ	. . .	3.03284								
Log M	. . .	4.33452	4.63544	4.81157	5.11260	5.28869	5.41363	5.58972	5.71466	5.81157
M	0° 0′ 0″	6° 0′ 3″	11° 59′ 56″	17° 59′ 59″	36° 0′ 0″	54° 0′ 0″	72° 0′ 0″	108° 0′ 0″	144° 0′ 0″	180° 0′ 0″
E	0° 0′ 0″	31° 11′ 50″	48° 20′ 30″	60° 13′ 0″	84° 24′ 38″	101° 38′ 30″	115° 47′ 44″	139° 33′ 18″	160° 21′ 15″	180° 0′ 0″
$e \sin E$	. . .	25° 11′ 47″	36° 20′ 30″	42° 13′ 0″	48° 24′ 40″	47° 38′ 30″	43° 47′ 40″	31° 33′ 20″	16° 21′ 14″	
$\frac{1}{2} E$	0° 0′ 0″	15° 35′ 55″	24° 10′ 15″	30° 6′ 30″	42° 12′ 19″	50° 49′ 15″	57° 53′ 52″	69° 46′ 39″	80° 10′ 37″	90° 0′ 0″
Log tan $\frac{1}{2} E$	. . .	9.44588	9.65206	9.76333	9.95757	0.08885	0.20248	0.43371	0.76158	
Log $\sqrt{\frac{1+e}{1-e}}$	. . .	0.54394								
Log tan $\frac{1}{2} v$	. . .	9.98982	0.19600	0.30727	0.50151	0.63279	0.74642	0.97765	1.30552	
$\frac{1}{2} v$	0° 0′ 0″	44° 19′ 45″	57° 30′ 40″	63° 45′ 46″	72° 30′ 31″	76° 53′ 19″	79° 50′ 8″	83° 59′ 24″	87° 10′ 1″	90° 0′ 0″
Log cos $\frac{1}{2} E$	. . .	9.98370	9.96015	9.93706	9.86967	9.80054	9.72544	9.53865	9.23199	
Log cos $\frac{1}{2} v$	. . .	9.85451	9.73008	9.64551	9.47794	9.35573	9.24668	9.01995	8.69396	
Log $\left(\frac{\cos \frac{1}{2} E}{\cos \frac{1}{2} v}\right)$	. . .	0.12919	0.23007	0.29155	0.39173	0.44481	0.47876	0.51870	0.53803	
Log $\left(\frac{\cos \frac{1}{2} E}{\cos \frac{1}{2} v}\right)^2$	. . .	0.25838	0.46014	0.58310	0.78346	0.88962	0.95752	1.03740	1.07606	
Log $a(1 - e)$	. . .	9.52384								
Log ρ	9.52384	9.78222	9.98398	0.10694	0.30730	0.41346	0.48136	0.56124	0.59990	0.61171
Log $\frac{2}{\rho}$	0.77719	0.51881	0.31705	0.19409	9.99373	9.88757	9.81967	9.73979	9.70113	9.68932
$\frac{2}{\rho}$	5.9867	3.3022	2.0752	1.5635	0.98567	0.77192	0.66019	0.54928	0.50249	0.48901
$\frac{1}{a}$	0.4521	. . .	. . .	. . .	0.45208					
$\frac{2}{\rho} - \frac{1}{a}$	5.5346	2.8501	1.6231	1.1114	0.53359	0.31984	0.20811	0.09720	0.05041	0.03693
Log $\left(\frac{2}{\rho} - \frac{1}{a}\right)$	0.7431	0.4549	0.2103	0.0459	9.7272	9.5049	9.3183	8.9877	8.7025	8.5674
Log (9322×10^6)	9.9695									
Log V^2	10.7126	10.4244	10.1798	10.0154	9.6967	9.4744	9.2878	8.9572	8.6720	8.5369
Log $V^{4.5}$	24.1033	23.4549	22.9045	22.5346	21.8176	21.3174	20.8975	20.1537	19.5120	19.2080

The second column of the following table contains the values of $V^{4.5}$, corresponding to the values of $t - T$, given in the first column. The third column contains a series of factors, so taken that if each value of $V^{4.5}$ is multiplied by the factor standing on the same line with it, the sum of all the products thus formed is approximately the mean value of $V^{4.5}$ for the whole orbit. The fourth column is sufficiently intelligible without further explanation.

$t - T$	$V^{4.5}$	Factor.	$V^{4.5} \times$ Factor.
0^d	$1\,268\,500 \times 10^{18}$	0.01667	$211\,400 \times 10^{17}$
20	285 000	.03333	95 020
40	80 260	.03333	26 750
60	34 250	.06667	22 830
120	6 571	.10000	6 571
180	2 077	.05000	1 039
240	790	.20000	1 580
360	142	.20000	285
480	33	.20000	65
600	16	.10000	16
			Sum $= 365\,556 \times 10^{17}$

Collecting our results, we have

$$\rho'' \sin s = 0.00019133 \qquad F = 8\,498 \times 10^{16}$$
$$a = 2.2141 \qquad V^{4.5} = 3\,656 \times 10^{19}$$
$$a' = 2.2097$$

Substituting these values in equation (9), it becomes

$$p = \frac{8\,612\,\sigma}{10^{16}}$$

by means of which Table IX has been computed, the values of σ being taken from Table VI.

TABLE IX.—*Density of the Supposed Resisting Medium in Space, expressed in terms of the Height in Inches of the Column of Mercury which it will support.*

[NOTE.—The numbers in the table are given in units of the fifteenth place of decimals.]

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	0.039 97	0.010 68	0.005 58	0.002 85
0.04	0.399 70	0.106 80	0.055 75	0.028 50
0.08	0.799 30	0.213 60	0.111 50	0.057 00
0.12	1.199 00	0.320 40	0.167 30	0.085 51

As the formulæ employed are open to some objections, these results can only be considered as indicating the probable *order* of density of the supposed resisting medium. This density comes out so slight, or rather the rarity of the medium comes out so great, that the figures fail to convey any definite idea to the mind, and we must have recourse to illustrations in order to bring out their significance. A man inhales, on an average, twenty cubic inches of air at each breath, or, in other words, a quantity of air equivalent in bulk to a large apple, (3.37 inches in diameter,) but if the density of the air were reduced to that of the supposed resisting medium, in order to live he would be obliged to take into his lungs, at each inspiration, a bulk of it equivalent to a sphere somewhere between fifteen and one hundred and seventeen miles in diameter. Again: let us suppose two little cylinders, the first having a diameter equal to the height of the column of mercury which the resisting medium will support, and the second having a diameter of 0.003 of an inch, which is that of an average human hair; then, if these two cylinders simultaneously commence growing, and both increase at precisely the same rate, when the first has attained the diameter of a human hair the second will have attained a diameter somewhere between fifteen times, and six thousand three hundred times that of the earth.

The best information in our possession seems to indicate that auroras seldom, if ever, appear at an elevation above the earth's surface less than about forty-five miles, and that they sometimes extend upward to an elevation of at least five hundred miles.[1] Let us compute the density of the atmosphere at these elevations, employing, for that purpose, Laplace's barometrical formula, modified in accordance with the most recent determinations. Assuming that at the earth's surface the height of the barometer is 30 inches, and the temperature 60° F., and at the point where the pressure is to be determined the temperature is — 200° F., substituting these values in the formula, neglecting insignificant terms, and reducing, we get

$$\text{Log } p = 1.4771 - \frac{x}{46\,530\left(1 + \frac{x + 52\,251}{20\,888\,629}\right)} \qquad (11)$$

where p is the pressure of the atmosphere, expressed in inches of mercury, at an altitude of x feet above the earth's surface. From this expression it results that, measuring upward from the earth's surface, at 45 miles the pressure of the atmosphere is $\frac{276}{10^6}$ of an inch of mercury, and, at 500 miles, it is $\frac{16}{10^{50}}$ of an inch of mercury. Now, if, in order to make the density of the resisting medium in space as small as possible, we suppose the comet to consist of gas at a temperature of 20 000° F., and a pressure equivalent to that just found for the atmosphere at 45 miles above the earth's surface, the formulæ already given will show that the mass of the comet must be 0.000 000 004 37 of that of the earth. The diameter of a sphere of lead of the same weight is 10.21 miles, and the density of the resisting medium in space must be such as would support $\frac{196}{10^{21}}$ of an inch of mercury. It thus appears that, with any value which can be rea-

[1] Professor E. Loomis on the Aurora Borealis, Smithsonian Report, 1865, p. 221.

sonably deduced for the mass of the comet, the density of the resisting medium comes out enormously greater than that of the atmosphere at the upper limit of auroras. Hence, there is some probability that the electric currents which give rise to auroras are propagated in a medium which pervades all space, and that the spectrum of the aurora is, in reality, the spectrum of that medium. This probability is strengthened by the fact that the characteristic line of the spectrum of the corona seen around the sun during total eclipses is sometimes visible in the spectrum of the aurora, and also by Professor C. Piazzi Smyth's recent discovery, that the spectrum of the nocturnal sky shows a maximum of brightness about the position which would be occupied by the coronal line if it were present.[1]

Without going into any elaborate discussion on the subject of comets' tails, I would like to call attention to the fact that many circumstances seem to indicate that they are, to a great extent, an electrical phenomenon. Such a view, taken in connection with what has been said in the preceding paragraph, would lead to the conclusion that, no matter how much the physical constitutions of the heads of *large* comets may vary, their tails must all give spectra similar to that of the aurora, although additional lines may be present. If future observations should confirm this conjecture, it seems to me that the existence of a resisting medium pervading all space will be proved.

IV.—SUMMARY OF RESULTS.

In conclusion, the results arrived at in this report may be briefly recapitulated as follows:

1. Encke's Comet gives a carbon-spectrum.

2. From November 18 to December 2 the wave-length of the brightest part of the second band of the comet's spectrum was continually increasing.

3. No polarization was detected in the light of the comet.

4. The mass of Encke's Comet is certainly not less than that of an asteroid.

5. The density of the supposed resisting medium in space, as computed from the observed retardation of Encke's Comet, is such that it would support a column of mercury somewhere between $\frac{220}{10^{17}}$ and $\frac{285}{10^{20}}$ of an inch high.

6. There is some probability that the electric currents which give rise to auroras are propagated in a medium which pervades all space, and that the spectrum of the aurora is, in reality, the spectrum of that medium.

7. It is not improbable that the tails of all *large* comets will be found to give spectra similar to that of the aurora, although additional lines may be present.

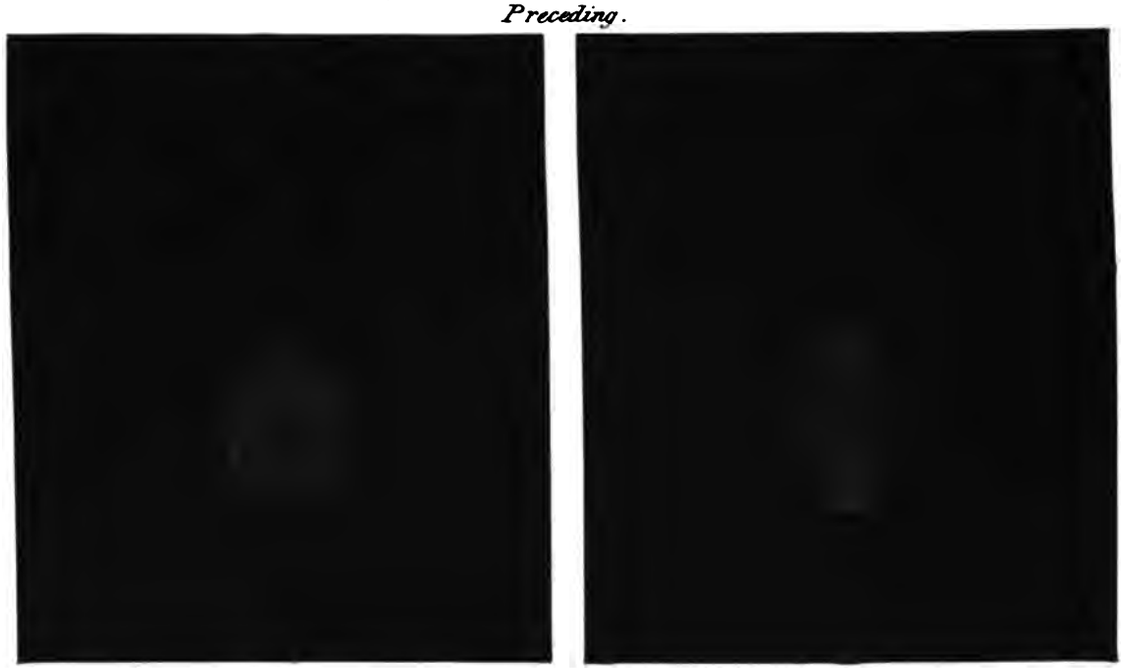
Very respectfully,

WM. HARKNESS,
Professor of Mathematics, U. S. Navy.

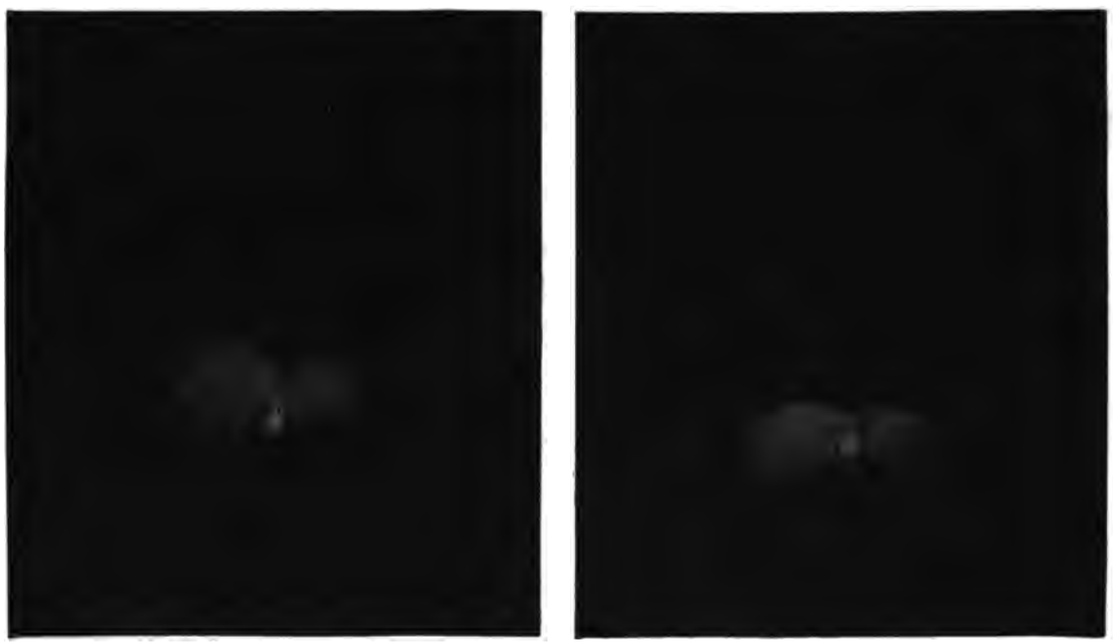
Rear-Admiral B. F. SANDS, U. S. N.,
Superintendent U. S. Naval Observatory, Washington, D. C.

[1] Proceedings of the Royal Society, 1872, Vol. XX, p. 136.

Preceding.



North. OCTOBER 17. NOVEMBER 17. *South.*



DECEMBER 1. *Following.* DECEMBER 2.

ENCKE'S COMET

as seen during its return in 1871

by

Prof. A. Hall, U. S. N.